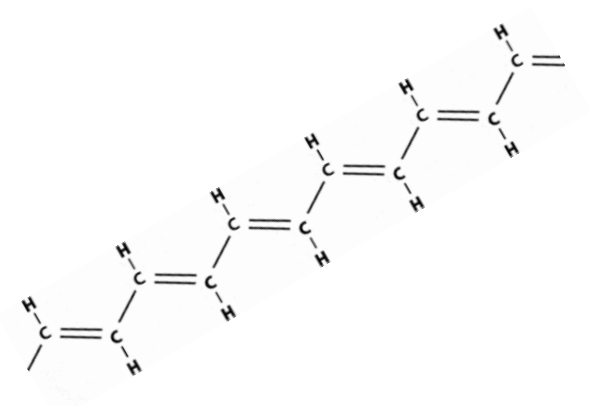
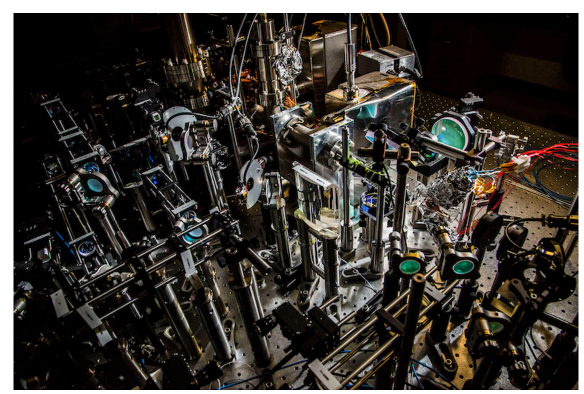
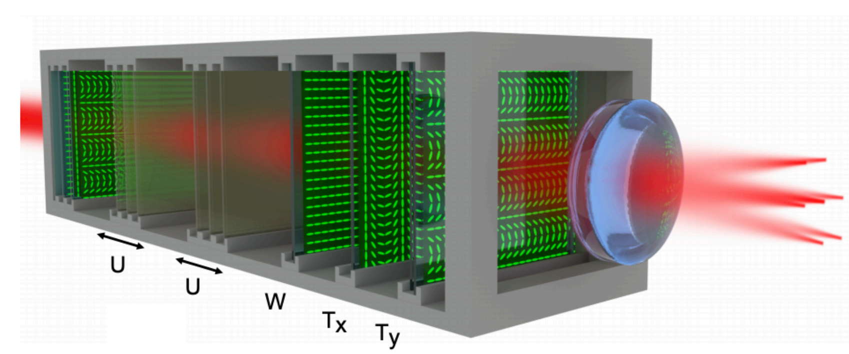
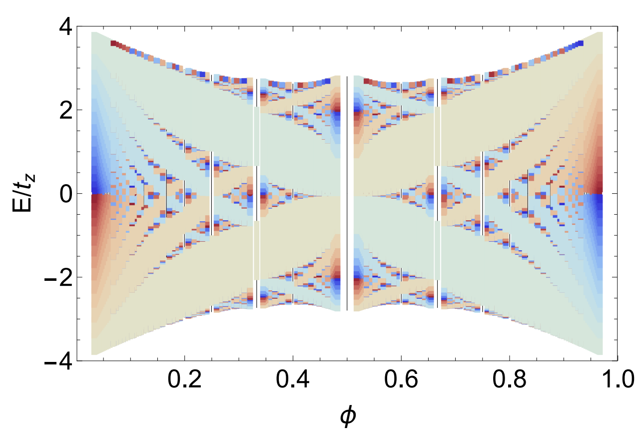
Topological phases in condensed matter and photonic systems

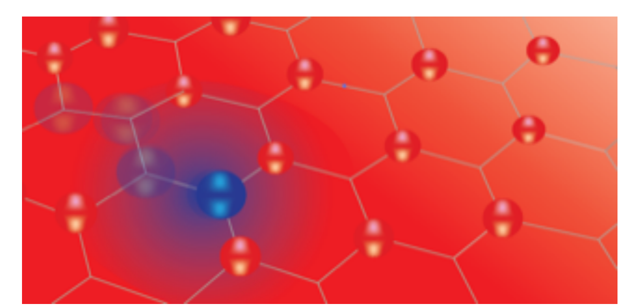
Pietro Massignan











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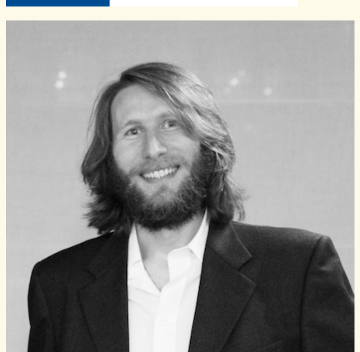
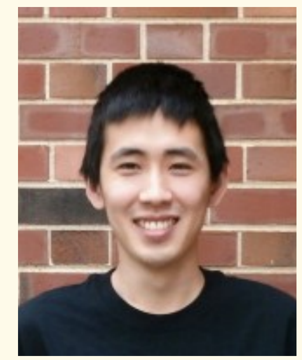


Nathan Goldman

Experiments

atomic wires

Eric J. Meier



Fangzhao An

PHYSICS ILLINOIS

Bryce Gadway

Hughes Taylor

photonics

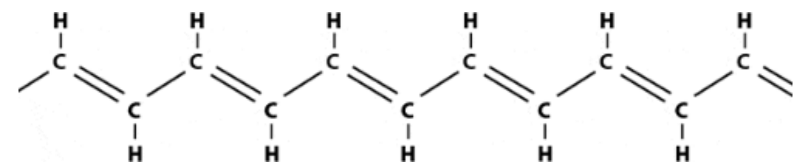
Alessio D'Errico

Filippo Cardano

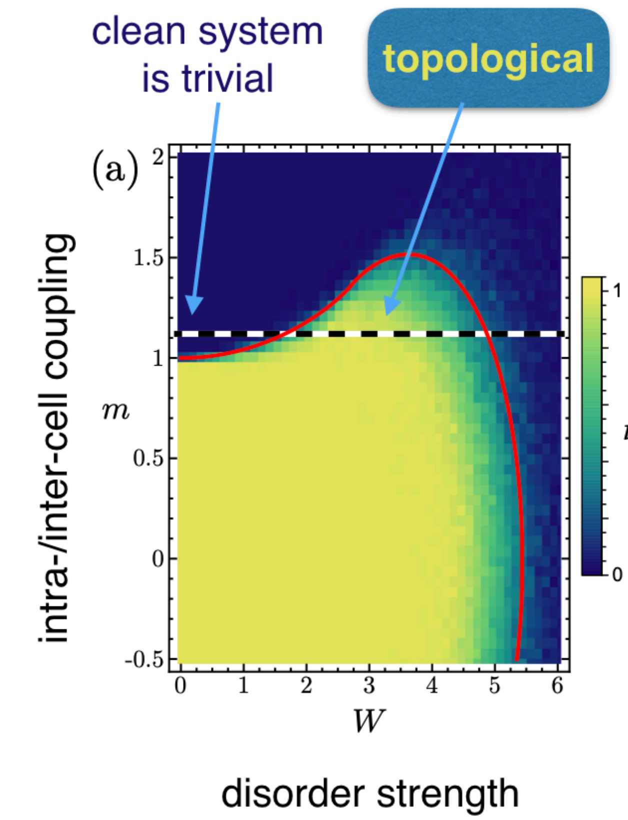
SLAM group

Lorenzo Marrucci

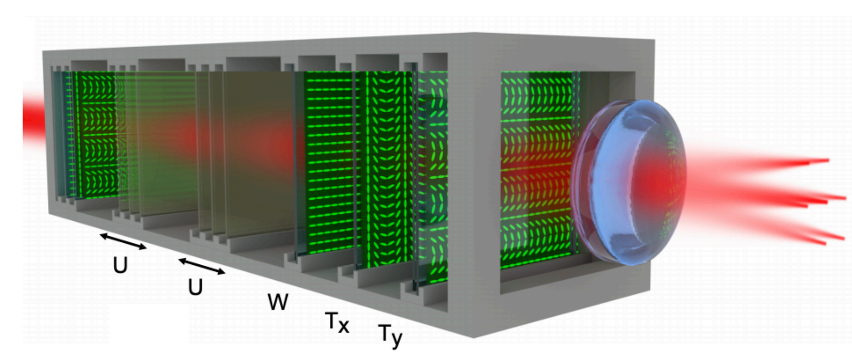
Outline



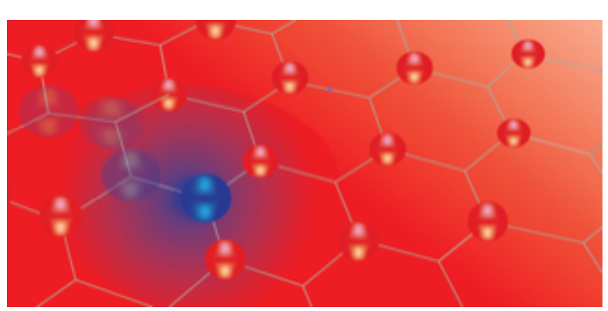
- Introduction



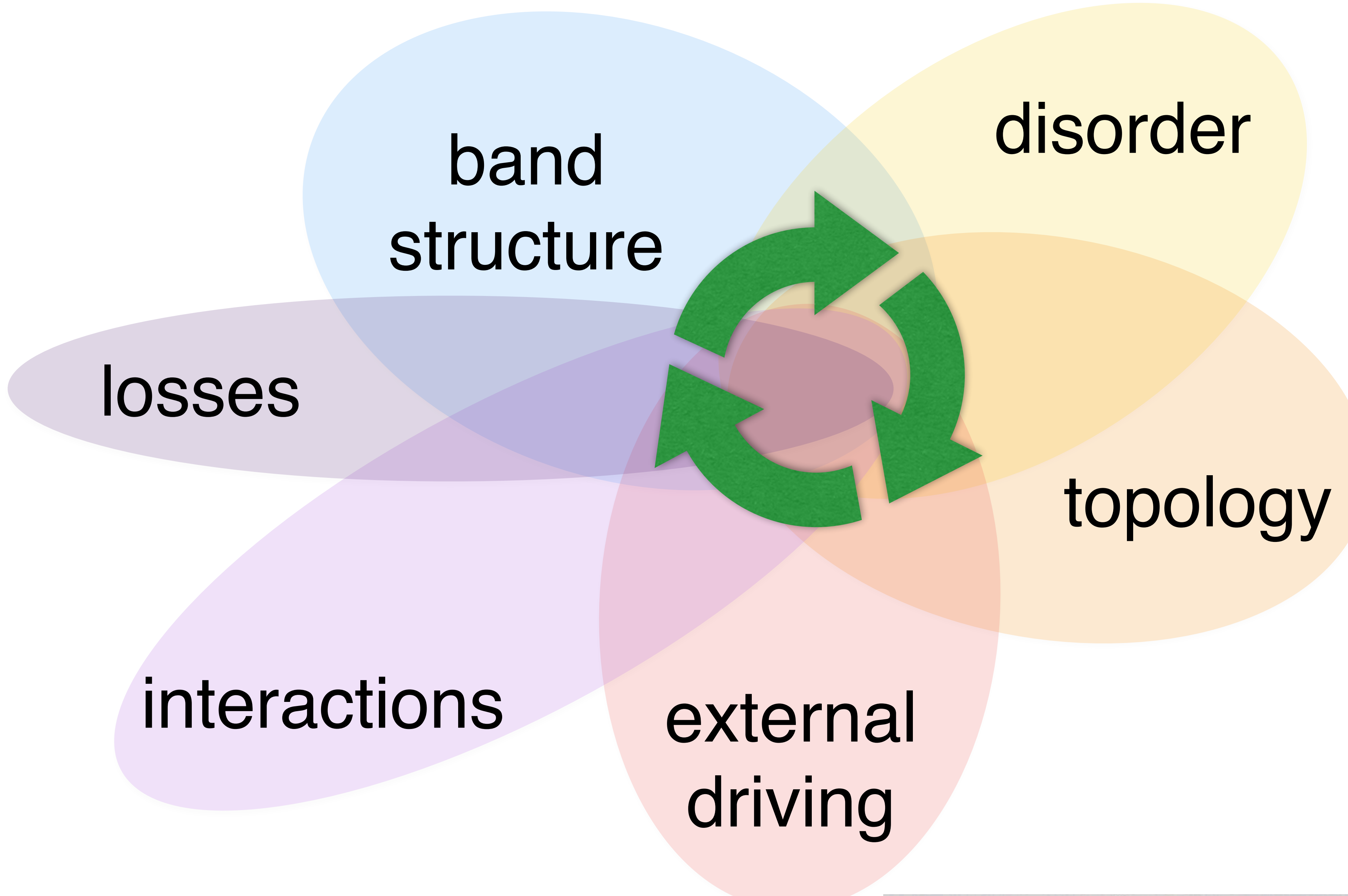
- One-dimensional chiral models
- Topological Anderson Insulator
- Chern insulator in a photonic quantum walk



- An impurity in a Chern insulator



Condensed matter



Plenty of emergent phenomena!

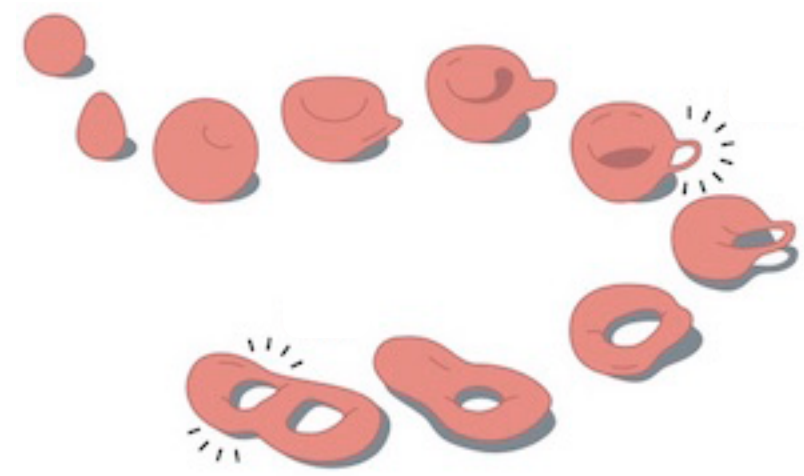
But we need to *observe* these.

E.g., how to “detect topology”?

Topology

- Classification of objects and manifolds under continuous deformations

  ✓ stretch and bend
  ✘ but don't cut, puncture, or glue

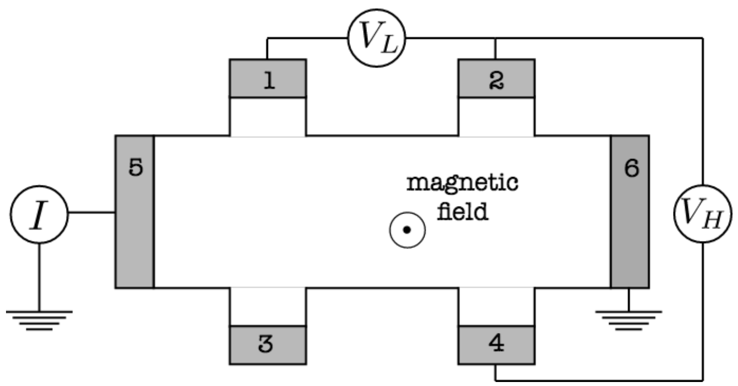


- *Global* properties!

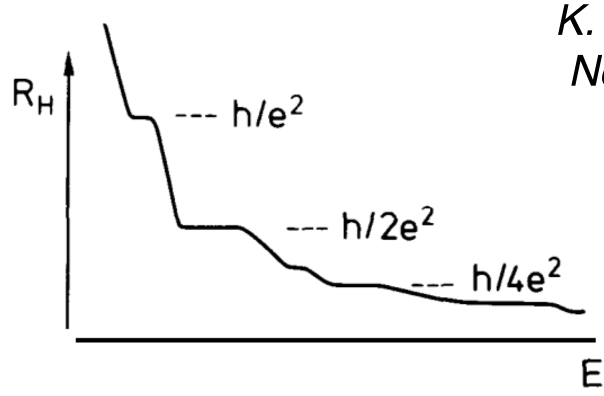
- Genus: number of holes

- Winding of a closed path
  (# of times it encircles a given point, line, …)

Hall effect



- Classical Hall effect (1879):
  when current flows in a 2D material,
  in presence of an out-of-plane B field,
  there appears a transverse (Hall) current



K. Von Klitzing
Nobel lecture

- Quantum Hall effect (1980):
  at low temperatures and high-B,
  the Hall current is quantized!

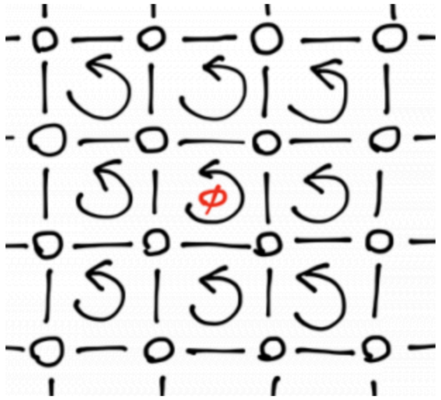
- Laughlin (1982): robustness due to **topology**

- TKNN (1982): Kubo formula links conductivity to *Chern numbers*
  (topological invariants defined on the occupied bands).

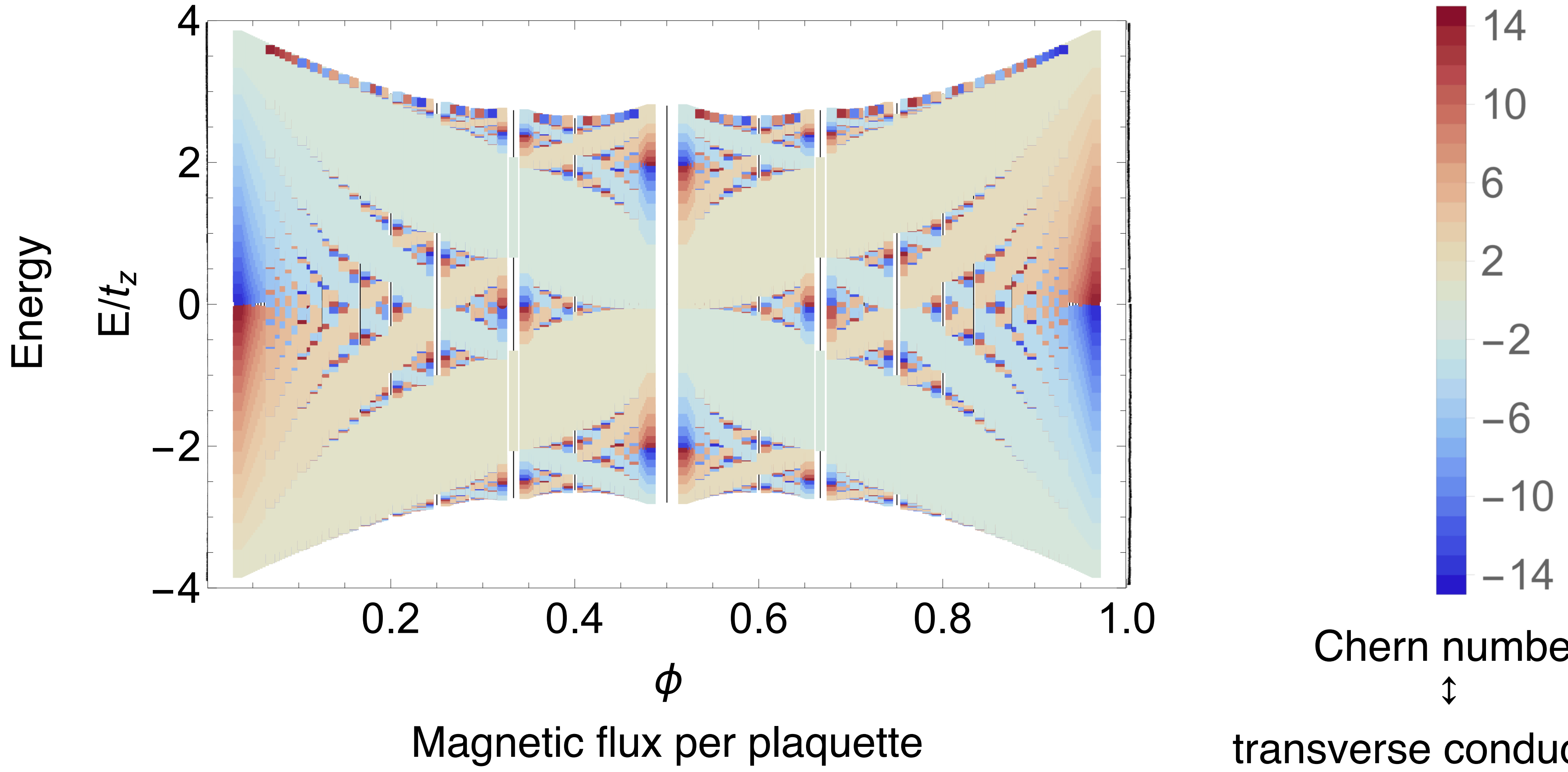
Thouless, Kohmoto, Nightingale & den Nijs
Phys. Rev. Lett. (1982)

Hofstadter butterfly

A very simple problem
hosts
a *fractal* spectrum



D. Hofstadter,
Phys. Rev. B (1976)



Chern number ν
↕
transverse conductivity $\nu e^2/h$

Topological insulators

- Insulators in the bulk, presenting robust current-carrying edge states
- Protected by the topology of bulk bands against local perturbations, like *disorder* and *defects*
- Enormous progresses in the last 10 years (QSH, 3D TIs., 4D QH, …)
- Characterization of non-interacting TIs in terms of discrete symmetries
  T: time-reversal
  C: charge-conjugation
  S: chiral

IQHE, Hofstadter, Chern insulators → A

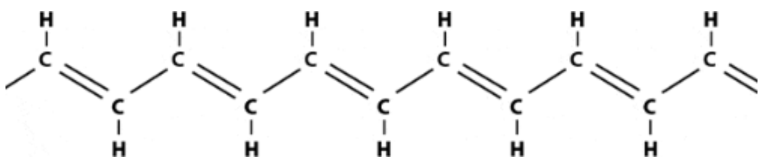
chiral → AIII, BDI

Class	T	C	S	0	1	2	3	4	5	6	7
				# of dimensions							
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	Chern number			$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
BDI	+	+	1	$\mathbb{Z}_2$	$\mathbb{Z}$	Winding		0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
D	0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
DIII	−	+	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
AII	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
CII	−	−	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
C	0	−	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Chiu, Teo, Schnyder & Ryu, Rev. Mod. Phys. (2016)

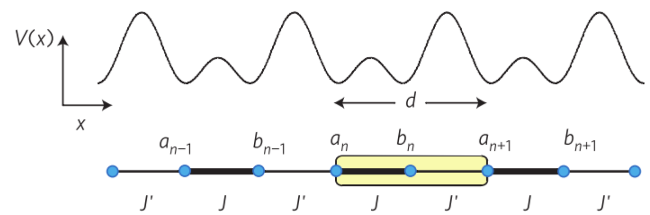
- Beyond the periodic table:
  Mott / crystalline / Anderson / Floquet TIs, …

1D chiral systems



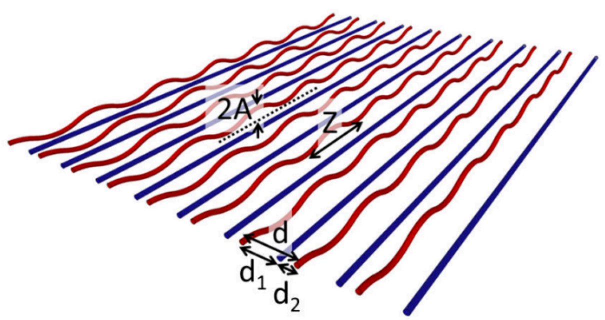
polyacetilene

[Nobel prize in Chemistry 2000]



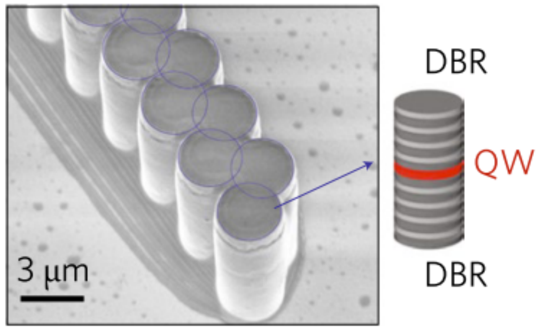
ultracold atoms
in superlattices

[M. Atala *et al.*, Nature Phys. 2013]



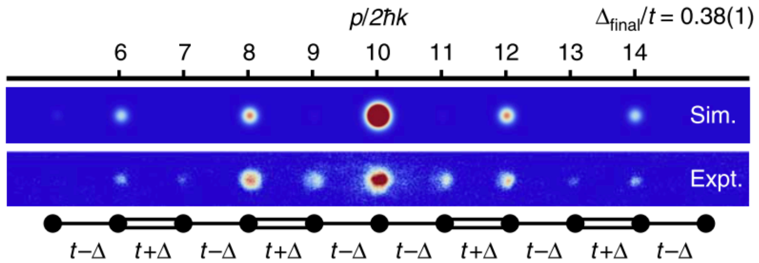
optical waveguides

[Zeuner *et al.*, PRL 2015]



cavity polaritons

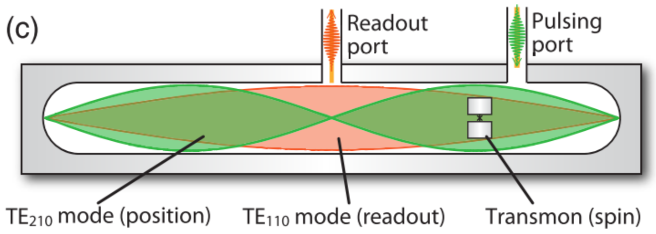
[St. Jean *et al.*, Nature Phot. 2017]



ultracold atoms
in momentum-lattices

[Meier *et al.*, Nature Comm. 2016]

[Meier, PM *et al.*, Science 2018]



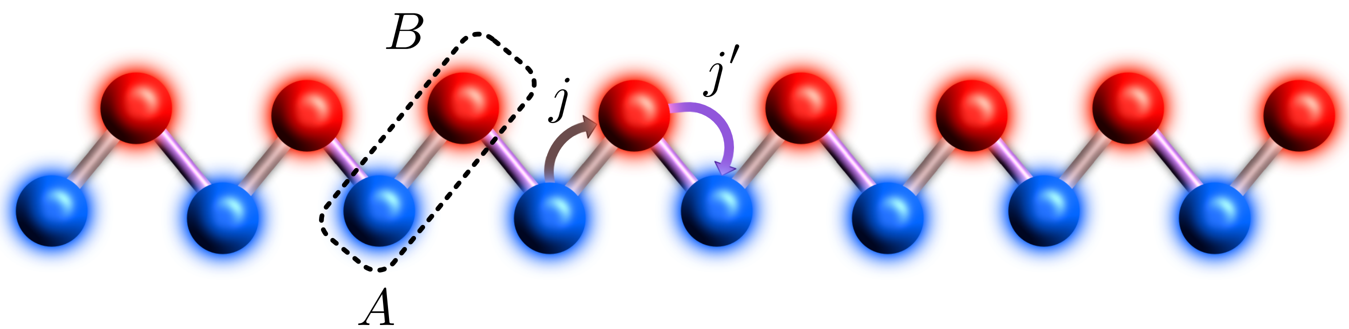
SC qubits
in mw-cavities

[Flurin *et al.*, PRX 2017]

SSH model

- Spinless fermions with staggered tunnelings:

Su, Schrieffer & Heeger
Phys. Rev. Lett. (1979)



Asbóth, Oroszlány, & Pályi
Lecture Notes in Physics (2016)

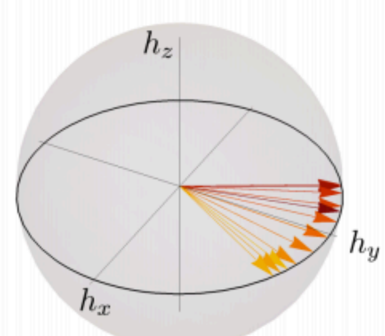
- ∃ two sublattices
  ∃ a "canonical basis" where H is purely off-diag: $$H = \begin{pmatrix} 0 & h^\dagger \\ h & 0 \end{pmatrix}$$

- Chiral symmetry: $\Gamma H \Gamma = -H$ (Γ: unitary, Hermitian, local)

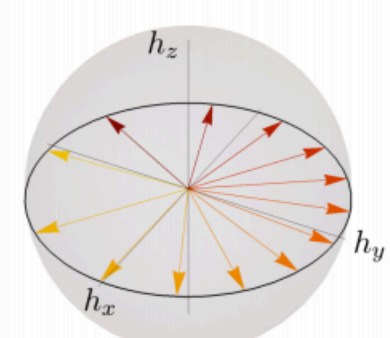
- In momentum space: $H_k = E_k \mathbf{n}_k \cdot \boldsymbol{\sigma}$ with $\mathbf{n}_k \perp \hat{\mathbf{z}} \quad \forall k$

$\Gamma = \sigma_z$

- Winding:



$\mathcal{W} = 0$



$\mathcal{W} = 1$

The winding *W*

- $\mathcal{W}$ may be calculated:

$$H_k = E_k \mathbf{n}_k \cdot \boldsymbol{\sigma}$$

  - from n : $\mathcal{W} = \oint \frac{\mathrm{d}k}{2\pi} \left(\mathbf{n} \times \partial_k \mathbf{n}\right)_z$
  - from the *eigenstates* : $\mathcal{W} = \oint \frac{\mathrm{d}k}{\pi} \mathcal{S},$ $\mathcal{S} = i\langle \psi_+ | \partial_k \psi_- \rangle$

skew polarization

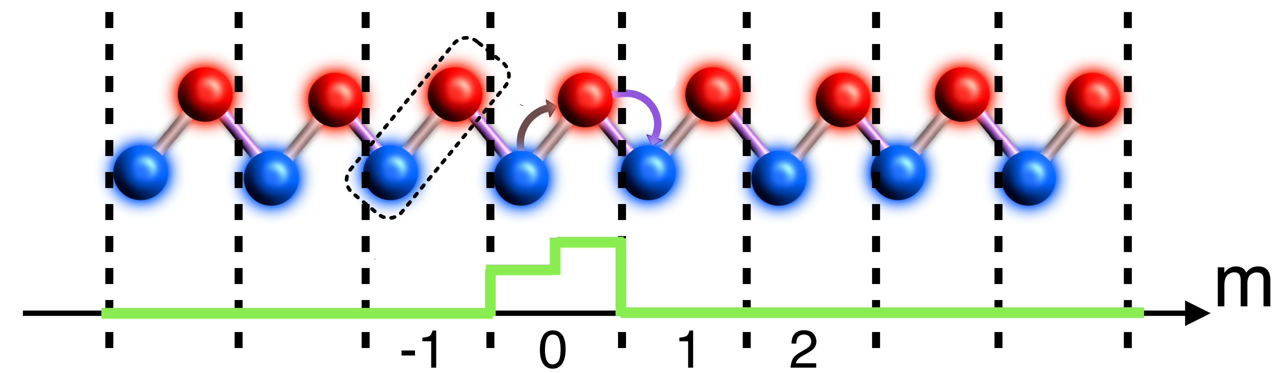
- What if the Hamiltonian is not known?
  Can one *measure* the winding?

  Yes, and it's simple!

Evolution in real time

- Initial condition

  localized on the $m\!=\!0$ cell:



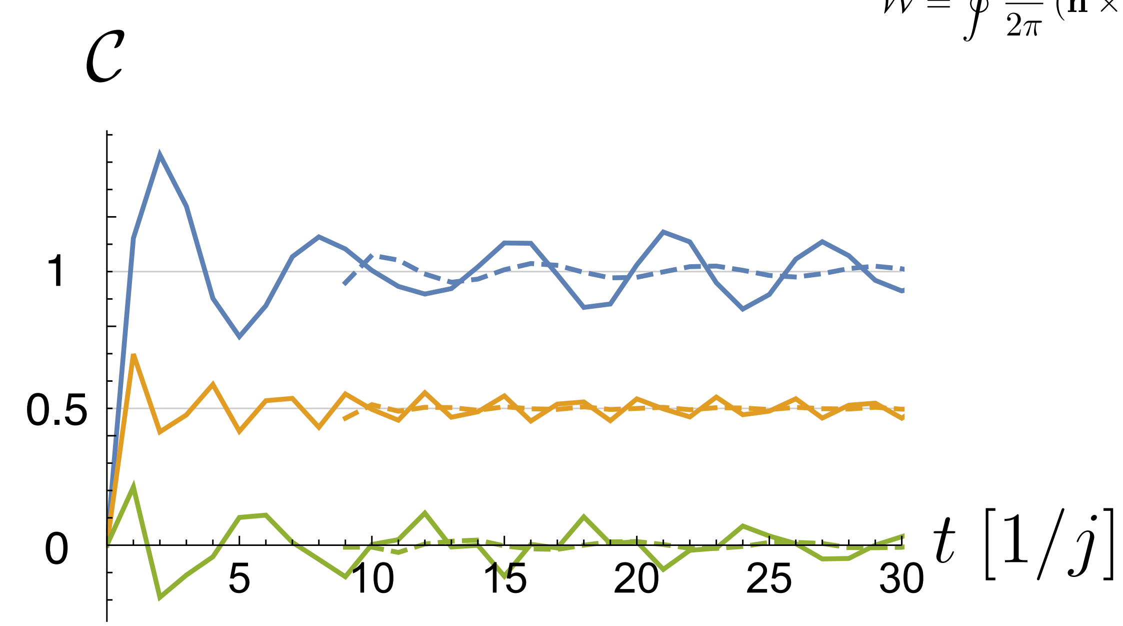
- **Mean Chiral Displacement:** $\mathcal{C}(t) \equiv 2\langle\widehat{\Gamma m}(t)\rangle = 2\Big[\langle m_A(t)\rangle - \langle m_B(t)\rangle\Big]$

$$\mathcal{C}(t) = 2\int_{-\pi}^{\pi}\frac{\mathrm{d}k}{2\pi}\,\langle U^{-t}\sigma_z(i\partial_k)U^t\rangle_{\psi_0} = 2\int_{-\pi}^{\pi}\frac{\mathrm{d}k}{2\pi}\sin^2(Et)\,|\mathbf{n}\times\partial_k\mathbf{n}| \xrightarrow{t\to\infty} \mathcal{W}$$

$$\mathcal{W} = \oint\frac{\mathrm{d}k}{2\pi}\,(\mathbf{n}\times\partial_k\mathbf{n})_z$$

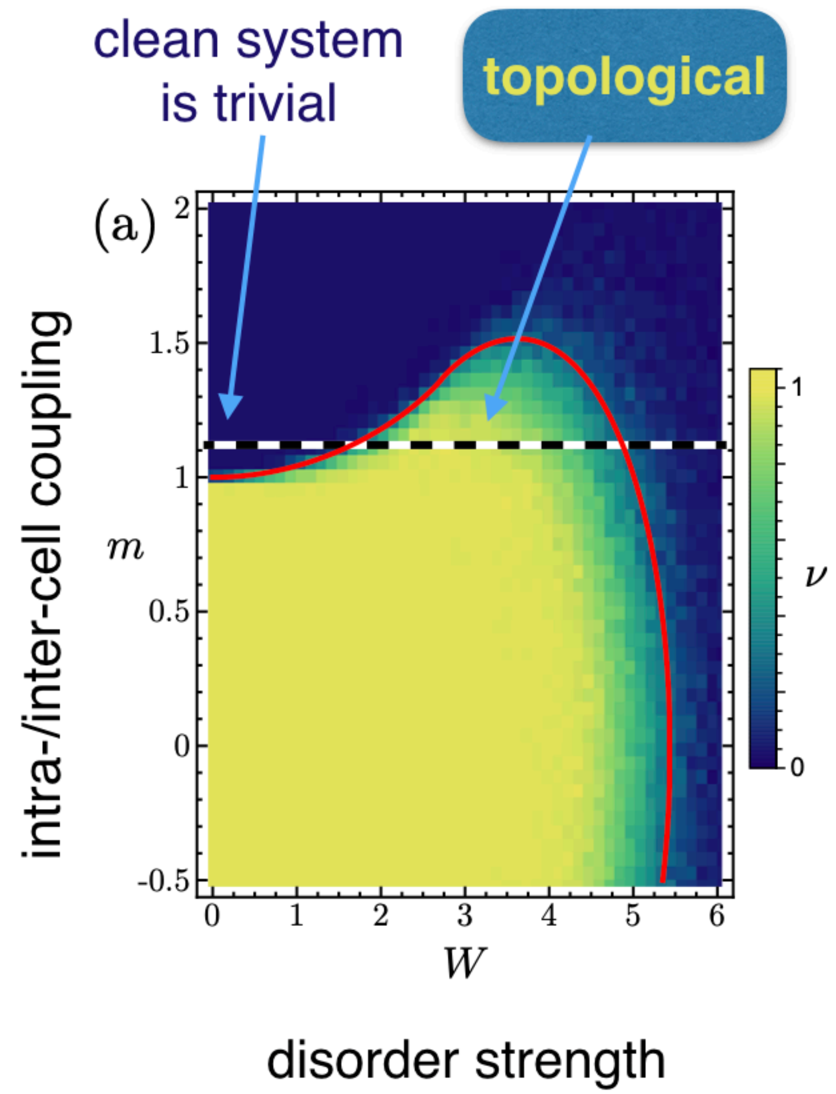
- *Bulk* measurement

- Fast convergence



Cardano, D'Errico, Dauphin, Maffei, … Marrucci, Lewenstein & PM
Nature Comm. (2017)

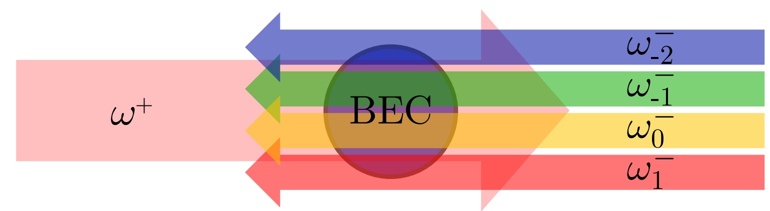
Topological Anderson insulator

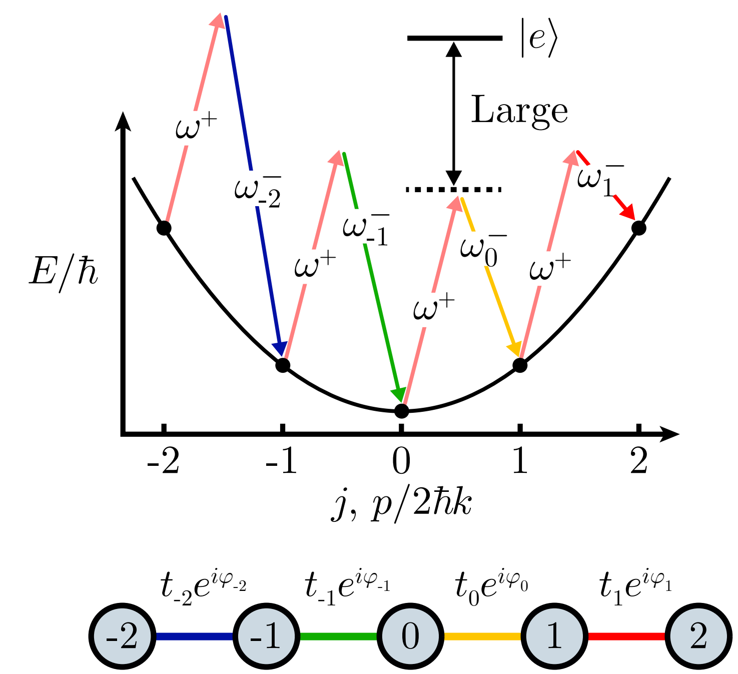


Meier, An, Dauphin, Maffei, PM, Taylor and Gadway,
Science (2018)

Atomic wires

- Atomic, non-interacting BEC





- Laser-driven coupling of discrete-momentum states

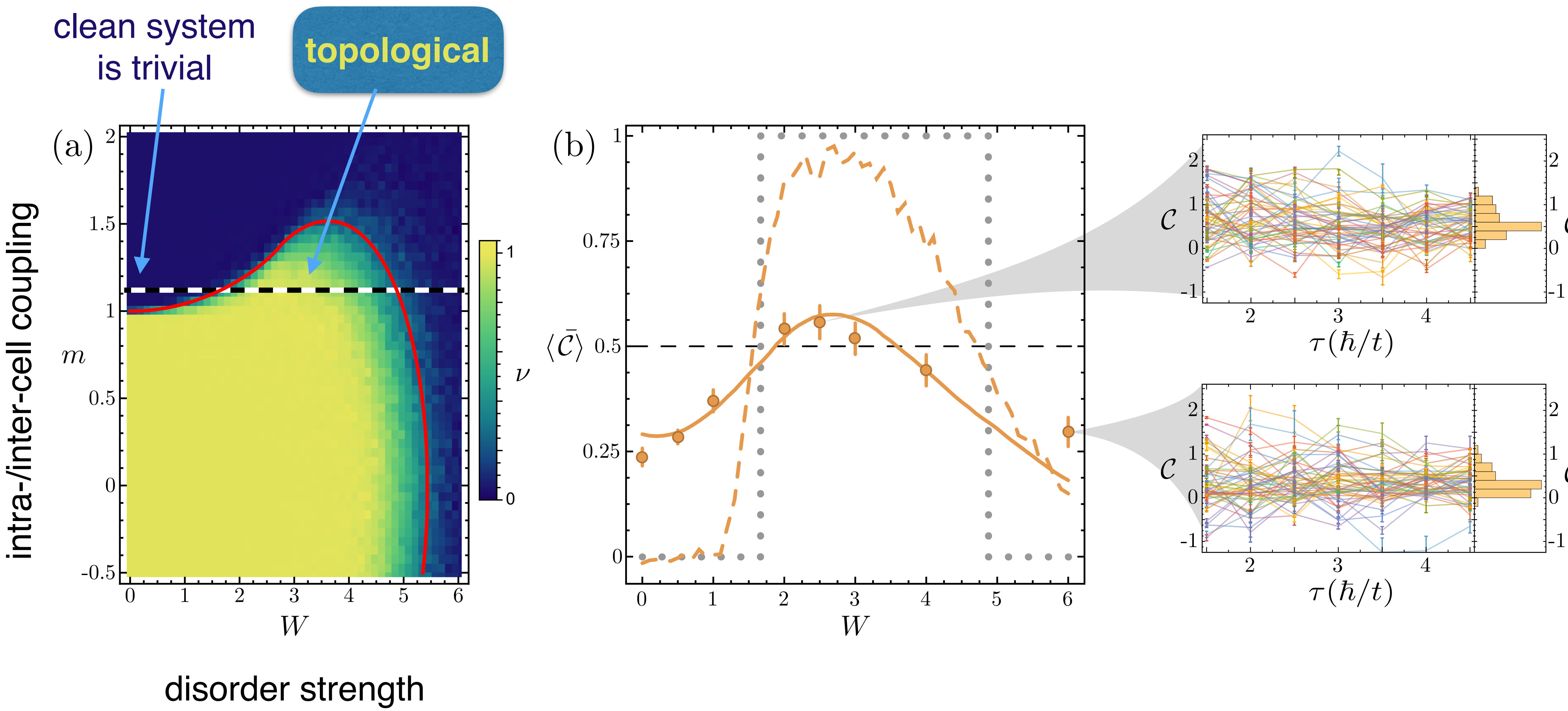
$$H_{\text{eff}} \approx \sum_j t_j (e^{i\varphi_j} |\widetilde{\psi}_{j+1}\rangle\langle\widetilde{\psi}_j| + \text{h.c.})$$

- 1D Hubbard model with full control on each tunneling's strength and phase

- Built-in chiral symmetry

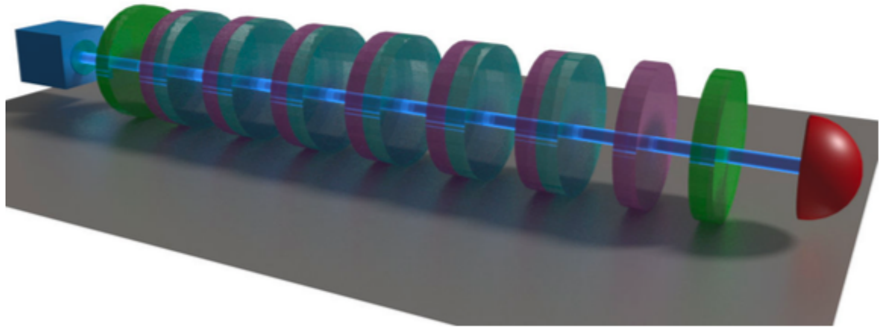
Topological Anderson transition

A trivial wire is driven into the topological phase by adding disorder

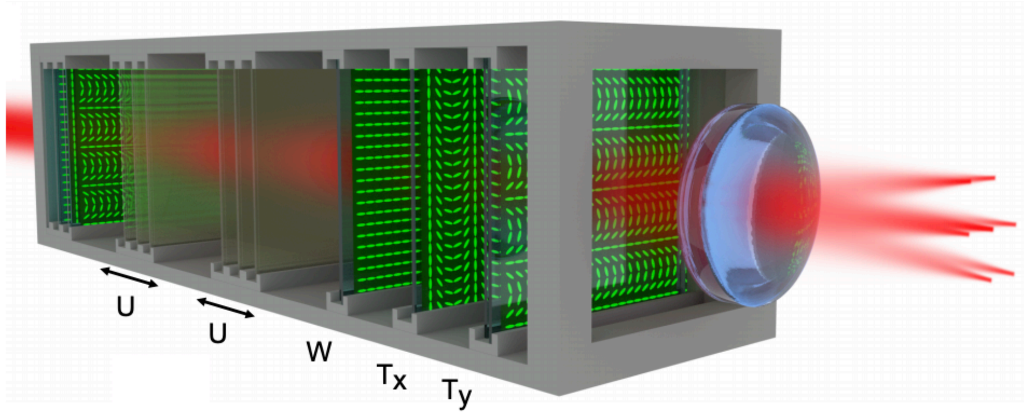


Meier, An, Dauphin, Maffei, PM, Taylor and Gadway,
Science (2018)

Photonic quantum walks

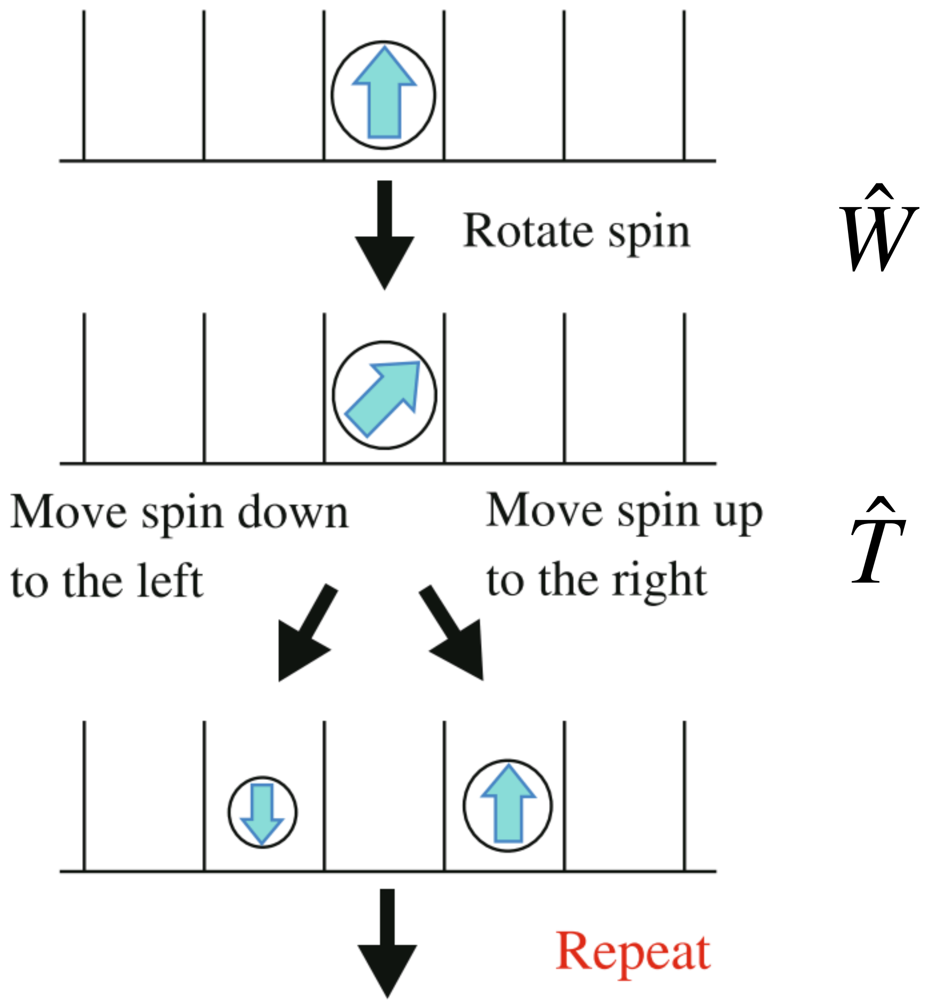


1D



2D

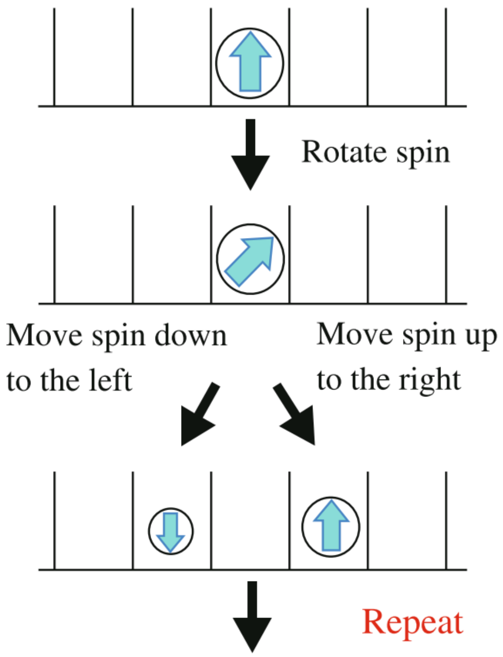
Quantum walk

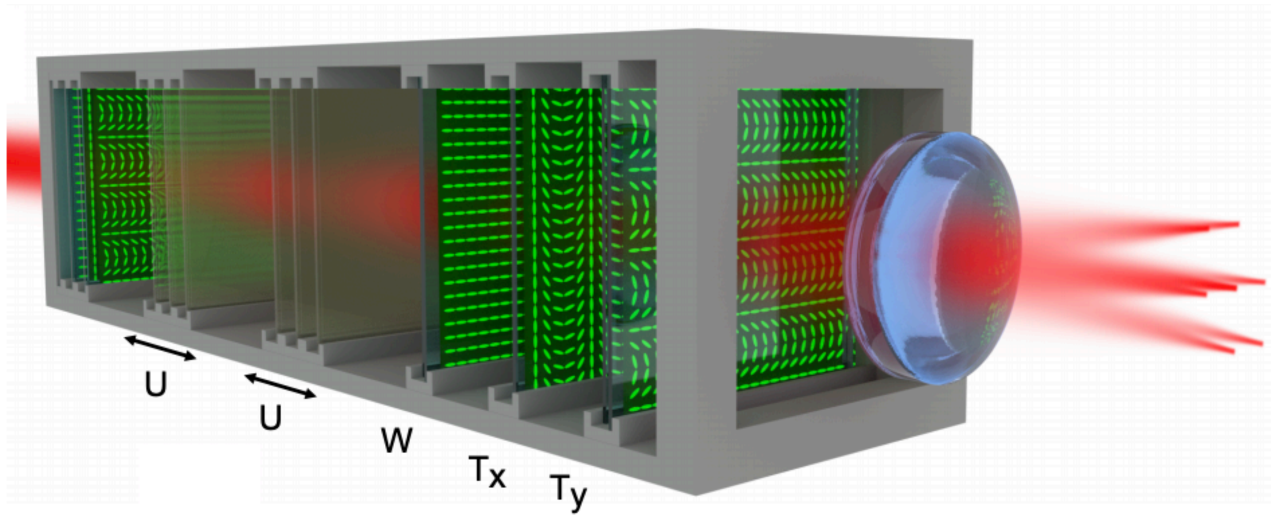


[Kitagawa, QIP 2012]

single step: $\hat{U} = \hat{T} \cdot \hat{W}$

2D Quantum Walk



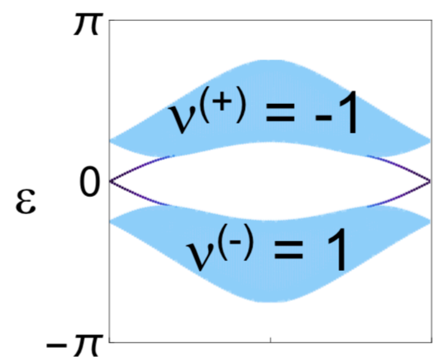


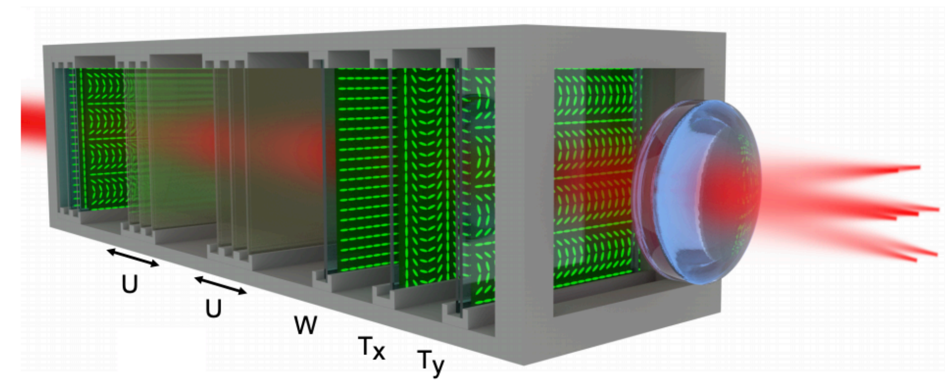
[D'Errico, PM *et al.*, arXiv 2018]

discrete-time QW	Experiment
walker's position	beam momentum in the transverse plane
spin state (↑/↓)	polarization (↻/↺)
spin rotation	quarter-wave plates (W)
conditional displacement	polarization-dependent diffraction gratings (T_x and T_y)
time	z-coordinate

2D walk: $\hat{U} = \hat{T}_y \cdot \hat{T}_x \cdot \hat{W}$

Transverse conductance





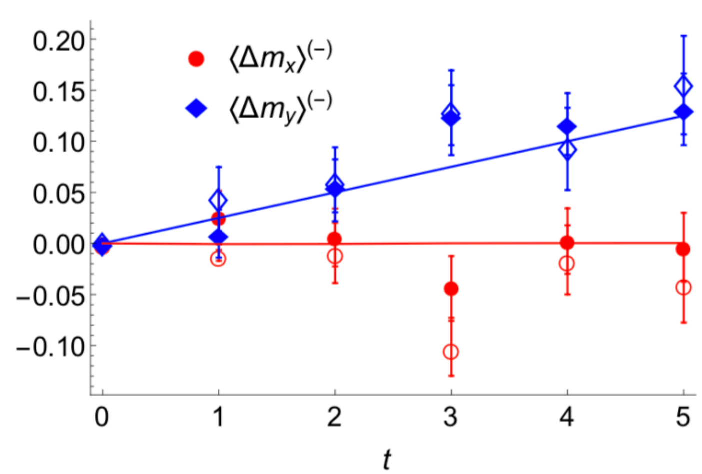
1) Choose efficiency of T_x and T_y which gives non-trivial topology

2) Prepare a tightly focused beam $\leftrightarrow$ a wavepacket in the lower band: $|\Psi(\mathbf{q}_0, -)\rangle$

3) Apply a force along x $\leftrightarrow$ $\mathbf{q}_\tau = (q_{0,x} + F_x\tau, q_{0,y})$ $\leftrightarrow$ displace the t^th^ T_x g-plate by $\Delta x_t \propto F_x t$

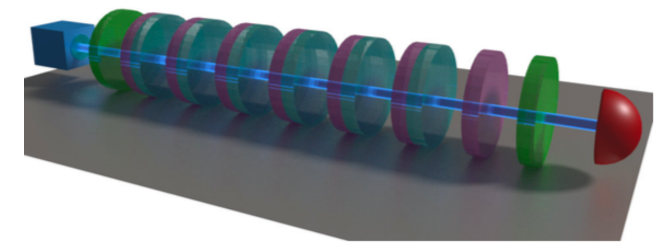
4) Transverse displacement of the beam after t steps: $\Delta m_y = \int_0^t \mathrm{d}\tau \left[v_y(\mathbf{q}_\tau) + F_x \Omega_{xy}(\mathbf{q}_\tau) \right] \approx \frac{F_x \nu^{(-)}}{2\pi} t$

Berry curvature

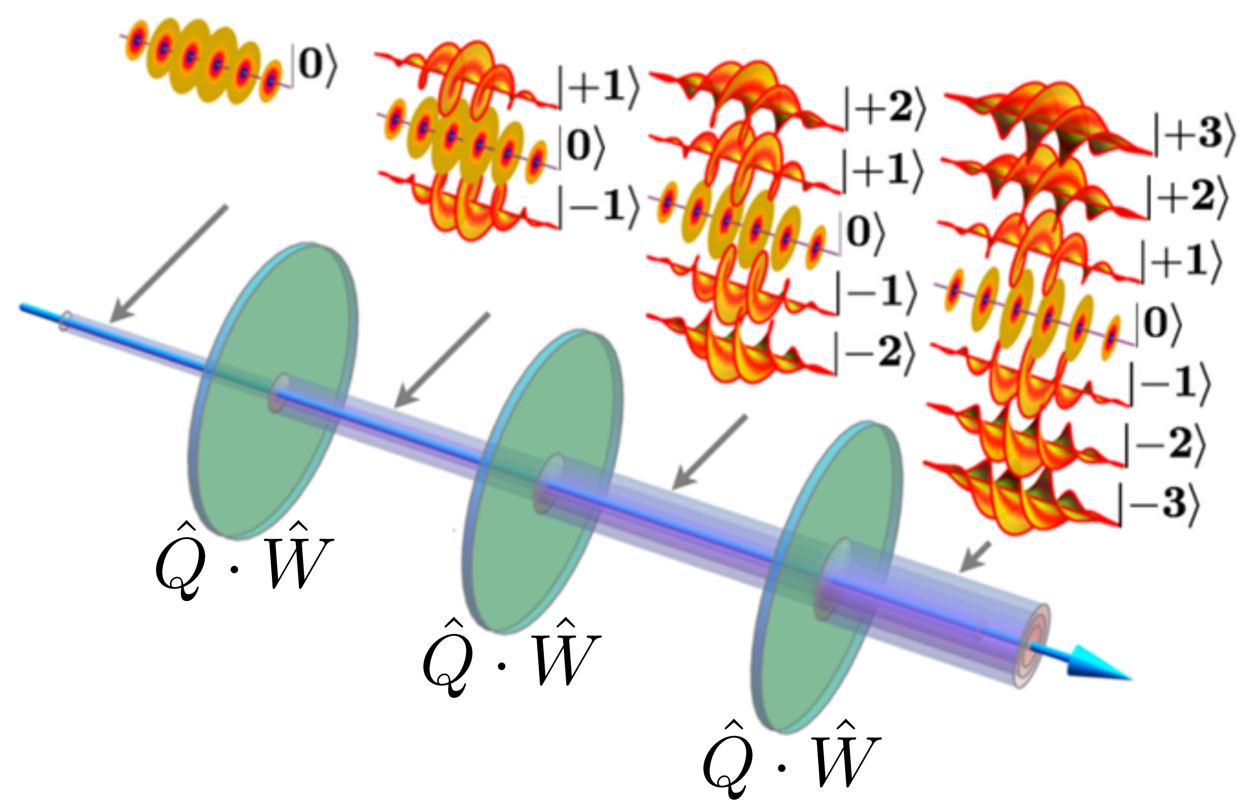


[D'Errico, PM *et al.*, arXiv 2018]

Quantum walk with twisted photons



discrete-time QW	Twisted photons
walker's position	orbital angular momentum (m)
spin state (↑/↓)	polarization (↻/↺)
spin rotation	quarter-wave plate (W)
conditional displacement	q-plate (Q)
time	z-coordinate

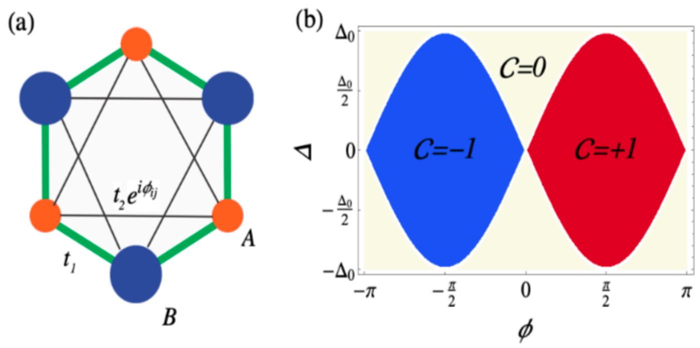


Topology of a periodically-driven system

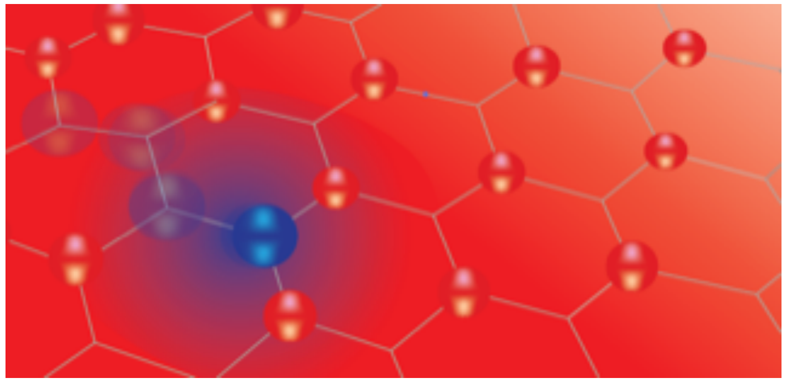
Cardano, D'Errico, Dauphin, Maffei, … Marrucci, Lewenstein & PM
Nature Comm. (2017)

Topological polarons

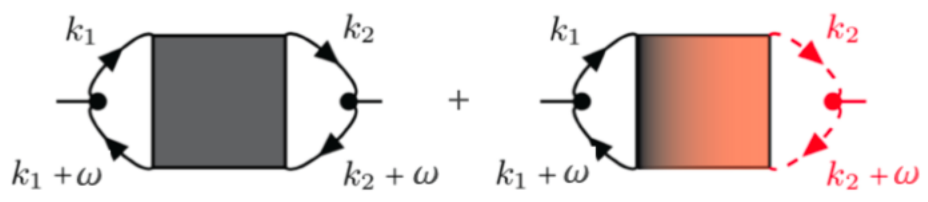
1) Prepare a half-filled Haldane insulator

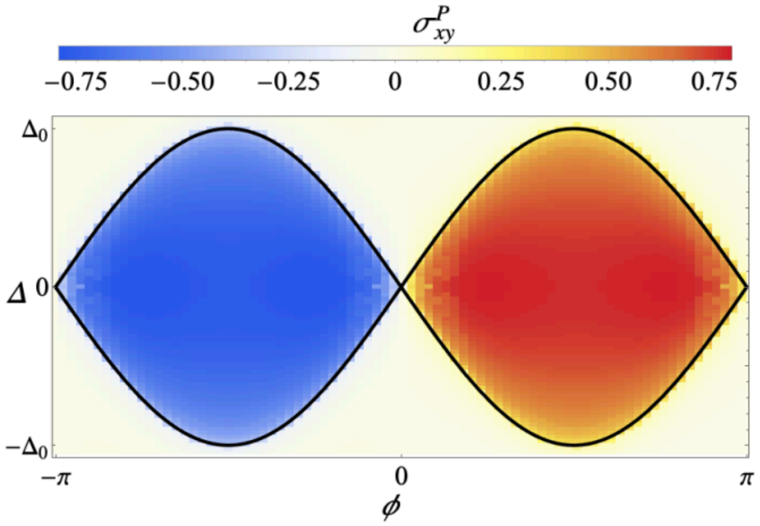


2) Dip an impurity in it to obtain a *topological polaron:* a quasiparticle dressed by a topological cloud



3) Evaluate the transverse conductivity of the polaron





[Camacho-Guardian, Goldman, PM and Bruun, *Rapid Comm. in PRB (2019)*]

Conclusions

- The *mean chiral displacement* captures the winding of 1D chiral systems (static, periodically driven, and disordered)
- Experimental observation of a **topological Anderson transition**
- Characterization of a periodically-driven 2D Chern insulator
- Quantum impurities (quasiparticles) in topological systems

- Dynamical observables for *other topological classes*?
- Interacting topological systems?

Cardano *et al.,* Nature Comm. 2017
Maffei *et al.,* New J. Phys. 2018
Meier *et al.,* Science 2018
Camacho-Guardian *et al.*, PRB (Rapid Comm.) 2019
D'Errico *et al.,* arXiv 2018

Thank you!

Resistance to disorder

SSH model in the topological phase $\quad j' = 2j \quad \rightarrow \quad \begin{cases} \mathcal{W} = 1 \\ \Delta_{\text{gap}} = 2j \end{cases}$

+

independent disorder of amplitude Δ on ***all*** tunnelings

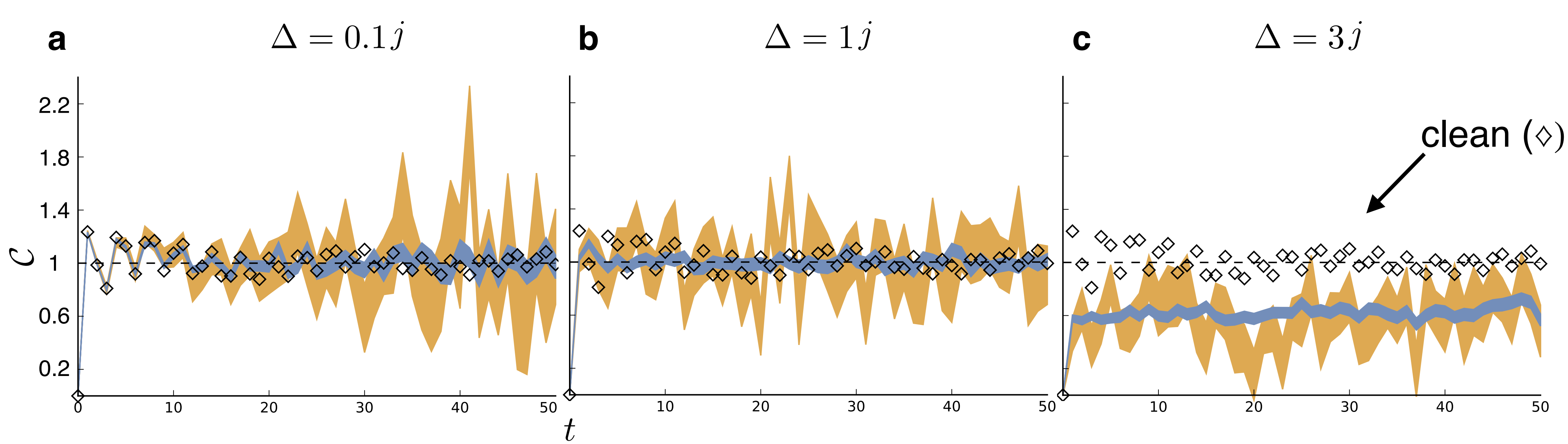
+

localized initial condition (randomly-polarized)

+

average over 50 (1000) disorder realizations

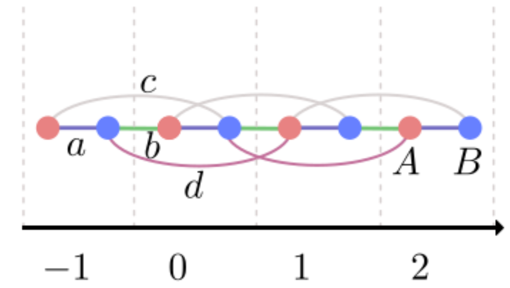
⬇

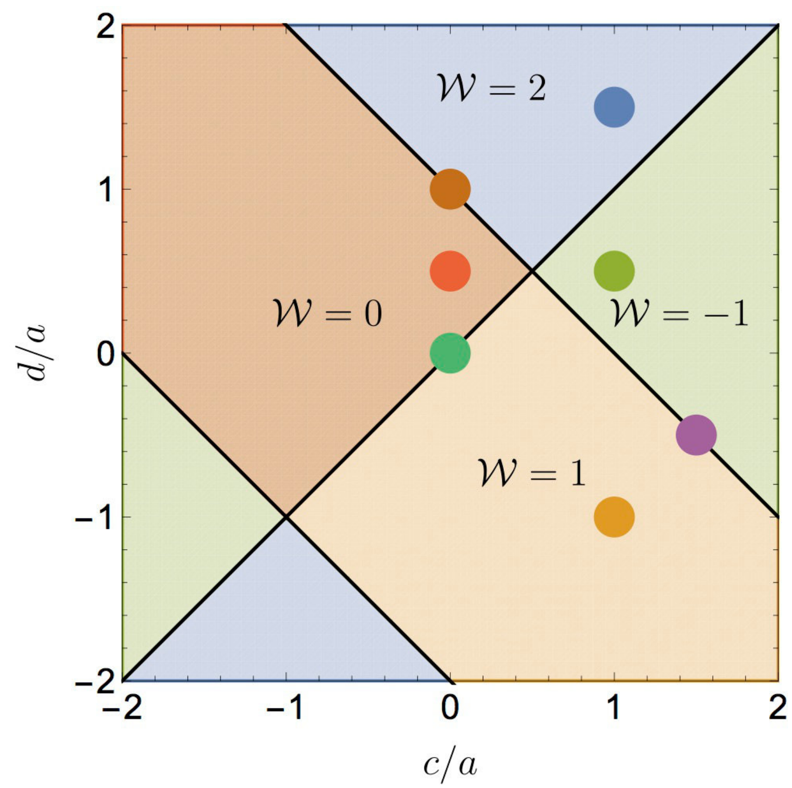


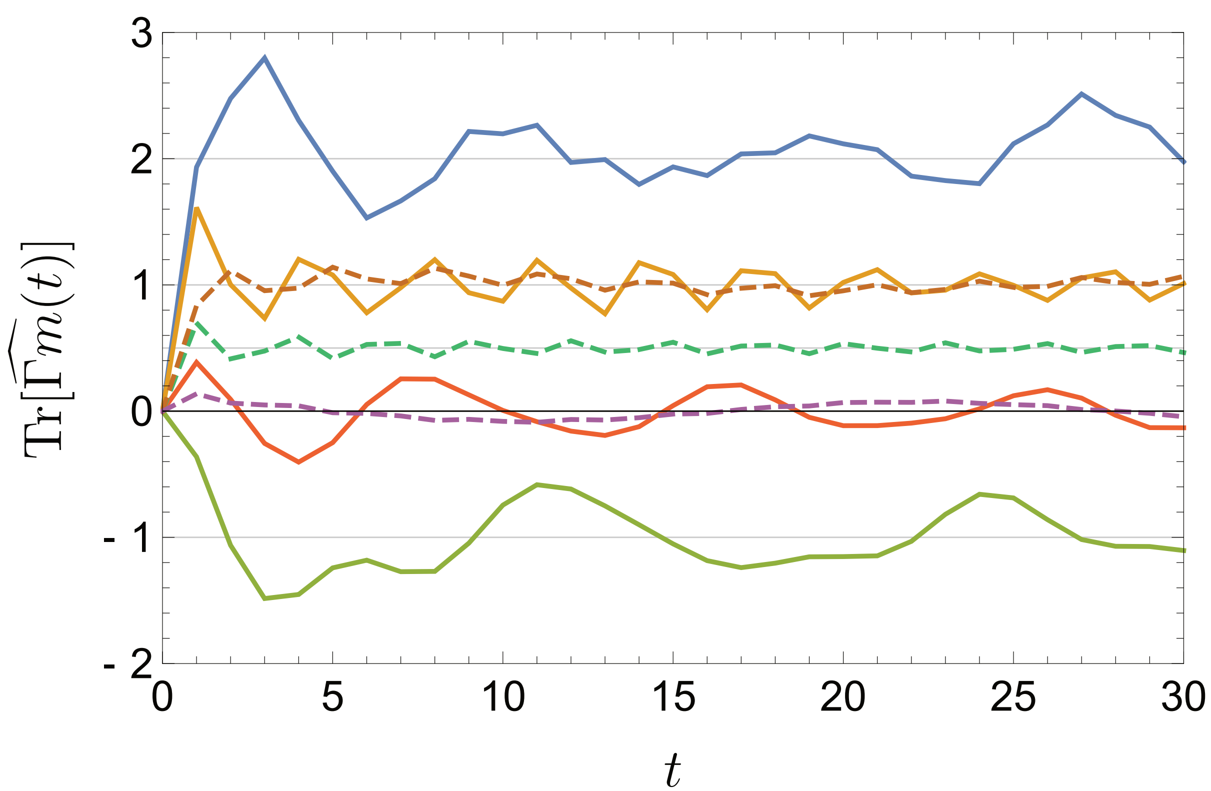
the MCD stays locked to the topological invariant as long as $\Delta < \Delta_{\text{gap}}$

Higher windings

- Extension to long-ranged models:







- At critical boundaries: MCD converges to the mean of the winding in the neighboring phases

Maffei, Dauphin, Cardano, Lewenstein & PM
New J. Phys. 2018